Linearity

Linearity is a measure of the bias change throughout the operating range of the gage. For example, consider the data in Table 1.

Table 1. Linearity example data.

Reference Value	Average Reading	Average Bias
12	11.98	-0.02
14	13.99	-0.01
16	16.00	0.00
18	18.01	0.01
20	20.02	0.02

The magnitude of the bias increases with the magnitude of the reference value. This bias is plotted as a function of the reference value in Figure 1. The slope of the plotted line, 0.005, is the linearity of the measurement system.

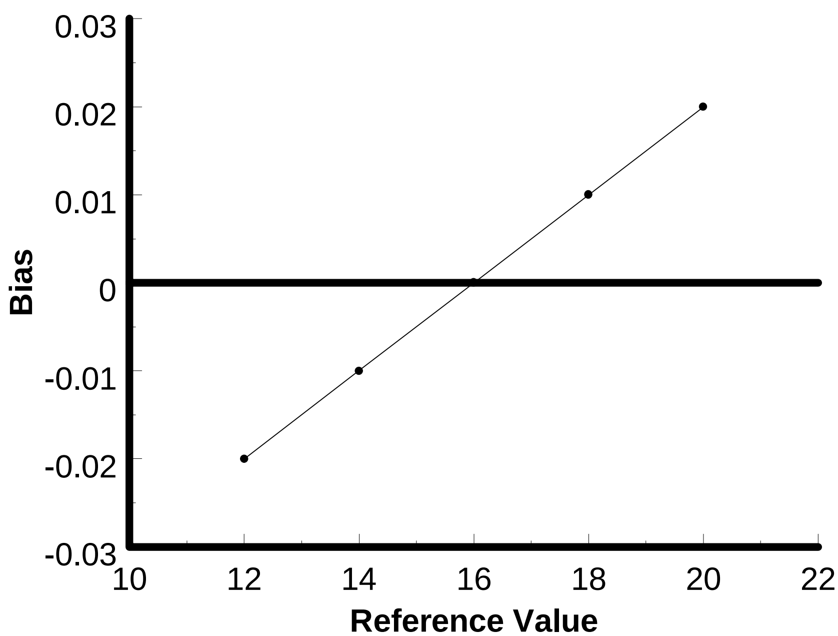

Figure 1. Linearity example.

The bias for the data shown in Table 2 is independent of the magnitude of the reference value. This data is plotted in Figure 2, and it is easy to see the slope is equal to zero; thus, the linearity is equal to zero.

Table 2. Linearity example data.

Reference Value	Average Reading	Average Bias
120	120.30	0.30
140	140.24	0.24
160	160.33	0.33
180	180.27	0.27
200	200.29	0.29

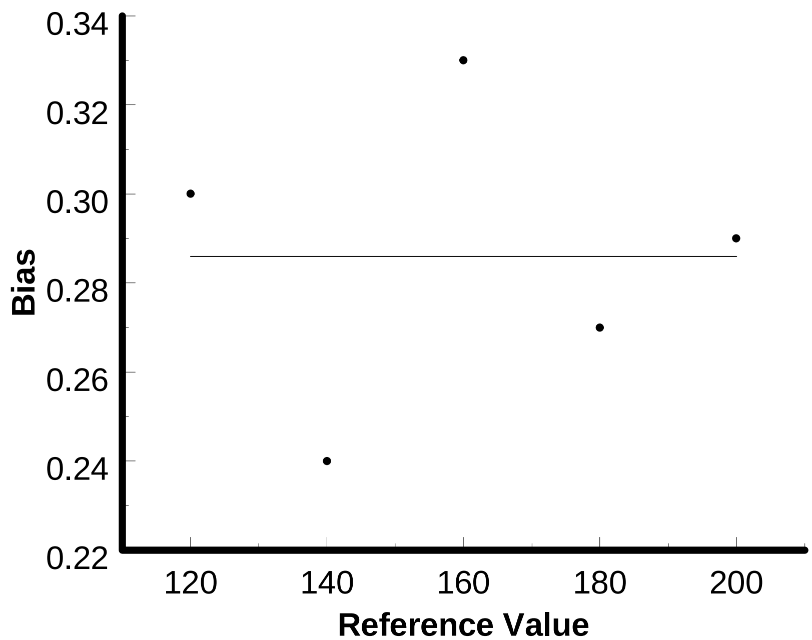

Figure 2. Linearity example.

Measurement system linearity is found by measuring reference part values throughout the operating range of the instrument, and plotting the bias against the reference values. The default procedure for determining linearity is to measure 10 parts 5 times each. The percent linearity is equal to the slope, *b*, of the best-fit straight line through the data points, and the linearity is equal the slope multiplied by the process variation:

$$L = bV_p \qquad (1)$$

The bias at any point can be estimate from the slope and the *y*-intercept, y_0 of the best-fit line:

$$B = y_0 + bx \qquad (2)$$

The estimated slope of the line, *b*, is

$$b = \frac{n\sum_{i=1}^{n} x_i y_i - \left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2} \qquad (3)$$

where n is the number of data points,
X_i is the ith reference value, and
Y_i is the ith bias.

The estimated y-intercept of the line is

$$y_0 = \frac{\sum_{i=1}^{n} y_i - b\sum_{i=1}^{n} x_i}{n} \quad 4$$

The coefficient of determination, r^2, is equal to the percentage of variability in the data set explained by the best-fit line.

$$r^2 = \frac{S_{xy}^2}{S_{xx} S_{xy}} \quad 5$$

where S_{xx} is the sum of squares of X,

$$S_{xx} = \sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n} \quad 6$$

and S_{yy} is the sum of squares of Y,

$$S_{yy} = \sum_{i=1}^{n} y_i^2 - \frac{\left(\sum_{i=1}^{n} y_i\right)^2}{n} \quad 7$$

and, S_{xy} is the sum of squares of X and Y

$$S_{xy} = \sum_{i=1}^{n} x_i y_i - \frac{\left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)}{n} \quad 8$$

Both Lotus 123 and Microsoft Excel have linear regression routines that automate the task of computing the characteristics of the best-fit straight line. In Lotus 123, the slope is computed using the function @Regression(“X-range”, “Y-range”, 101), the *y*-intercept is computed using the function @Regression(“X-range”, “Y-range”, 1), and the coefficient of determination is computed using the function @Regression(“X-range”, “Y-range”, 3). In Microsoft Excel, the slope is computed using the function =Slope(“Y-range”, “X-range”), the *y*-intercept is computed using the function =Intercept(“Y-range”, “X-range”), and the coefficient of determination is computed by squaring the function =Correl(“Y-range”, “X-range”).

Example

Table 3 shows the results when 10 parts were measured 5 times each. The last column of this table shows the computed bias. The process standard deviation is 0.03.

Table 3. Linearity example data.

Reference	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Bias
10	9.91	9.71	9.96	9.69	9.90	-0.17
20	19.92	20.16	19.88	19.85	19.92	-0.05
30	29.68	30.15	30.46	29.48	30.15	-0.02
40	40.03	39.93	40.08	39.69	40.03	-0.05
50	50.31	49.78	50.04	49.86	50.06	0.00
60	60.58	59.34	59.61	59.88	60.66	0.01
70	70.14	69.93	69.52	69.86	70.85	0.06
80	79.10	79.93	79.83	80.78	80.60	0.05
90	89.40	89.40	90.47	89.96	91.39	0.12
100	100.91	99.49	100.24	100.67	99.42	0.15

Figure 3 shows the bias plotted versus the reference value. The slope of the best-fit line is 0.00288, and the *y*-intercept is -0.14733. The percent linearity is equal to the slope, 0.288%, and the linearity is equal to the slope multiplied by the process variation:

$$L = 0.00288(0.03) = 0.0000864$$

The bias at any value is estimated from the expression

$$B = -0.14733 + 0.002881x$$

where *x* is the reference value.

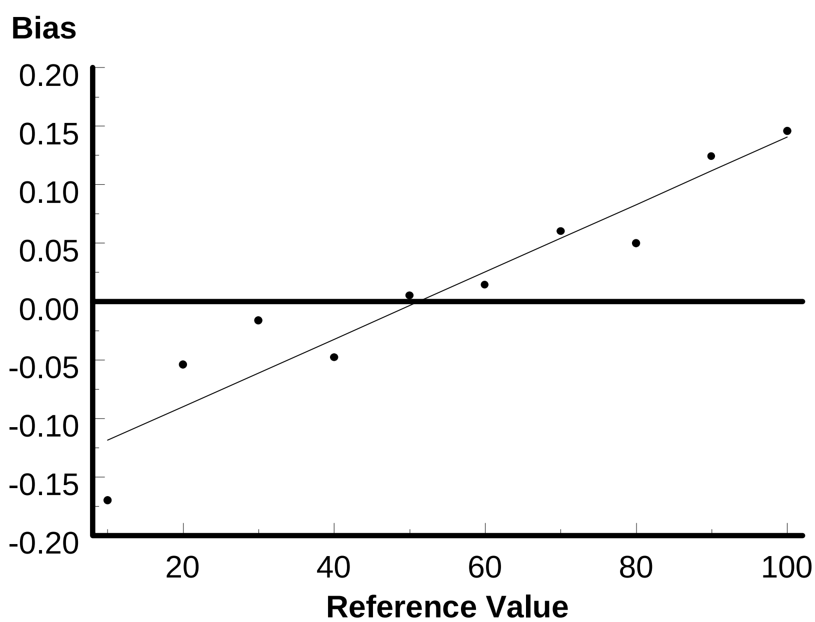

Figure 3. Linearity example.